Cambridge International **AS Level**

Cambridge International Examinations
Cambridge International Advanced Subsidiary Level

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **(P2)** **October/November 2016**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **3** printed pages, **1** blank page and **1** insert.

CAMBRIDGE International Examinations

[Turn over

1 The sequence of values given by the iterative formula

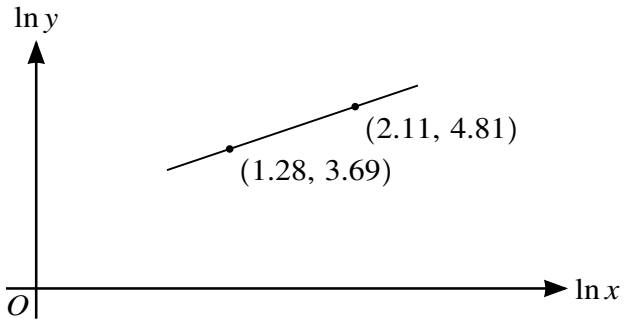
$$x_{n+1} = \frac{4}{x_n^2} + \frac{2x_n}{3},$$

with initial value $x_1 = 2$, converges to α.

(i) Use this iterative formula to find α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(ii) State an equation that is satisfied by α, and hence find the exact value of α. [2]

2



The variables x and y satisfy the equation $y = Kx^p$, where K and p are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points (1.28, 3.69) and (2.11, 4.81), as shown in the diagram. Find the values of K and p correct to 2 decimal places. [5]

3 **(i)** Find $\int \tan^2 4x \, \mathrm{d}x$. [2]

(ii) Without using a calculator, find the exact value of $\int_0^{\frac{1}{12}\pi} (4\cos 2x + 6\sin 3x) \, \mathrm{d}x$. [3]

4 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = ax^3 + 3x^2 + 4ax - 5,$$

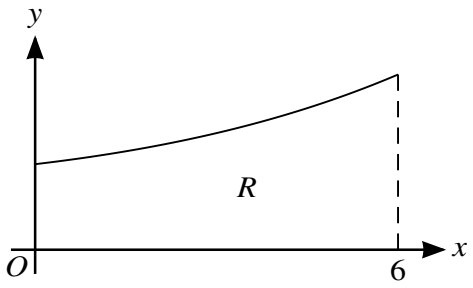
where a is a constant. It is given that $(2x - 1)$ is a factor of $\mathrm{p}(x)$.

(i) Use the factor theorem to find the value of a. [2]

(ii) Factorise $\mathrm{p}(x)$ and hence show that the equation $\mathrm{p}(x) = 0$ has only one real root. [4]

(iii) Use logarithms to solve the equation $\mathrm{p}(6^y) = 0$ correct to 3 significant figures. [2]

5



The diagram shows the curve $y = \sqrt{\left(1 + e^{\frac{1}{3}x}\right)}$ for $0 \leqslant x \leqslant 6$. The region bounded by the curve and the lines $x = 0$, $x = 6$ and $y = 0$ is denoted by R.

(i) Use the trapezium rule with 2 strips to find an estimate of the area of R, giving your answer correct to 2 decimal places. [3]

(ii) With reference to the diagram, explain why this estimate is greater than the exact area of R. [1]

(iii) The region R is rotated completely about the x-axis. Find the exact volume of the solid produced. [4]

6 A curve has parametric equations

$$x = \ln(t + 1), \quad y = t^2 \ln t.$$

(i) Find an expression for $\frac{dy}{dx}$ in terms of t. [5]

(ii) Find the exact value of t at the stationary point. [2]

(iii) Find the gradient of the curve at the point where it crosses the x-axis. [2]

7 **(i)** Express $\sin 2\theta(3 \sec \theta + 4 \operatorname{cosec} \theta)$ in the form $a \sin \theta + b \cos \theta$, where a and b are integers. [3]

(ii) Hence express $\sin 2\theta(3 \sec \theta + 4 \operatorname{cosec} \theta)$ in the form $R \sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Using the result of part **(ii)**, solve the equation $\sin 2\theta(3 \sec \theta + 4 \operatorname{cosec} \theta) = 7$ for $0° \leqslant \theta \leqslant 360°$. [4]

BLANK PAGE